A SYSTEMATIC ANALYSIS OF FREQUENCY DISTRIBUTIONS BY THE EDGEWORTH METHOD

BY

ERNEST EVRED BLANCHE

A.B., Bucknell University, 1938
A.M., Bucknell University, 1938

AN ABSTRACT OF A THESIS

SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY IN MATHEMATICS IN THE GRADUATE SCHOOL OF THE UNIVERSITY OF ILLINOIS, 1941

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The author is deeply indebted to Professor A. R. Crathorne for suggesting the problem, outlining the method of approach, and guiding it to a conclusion, and wishes at this time to express his sincere appreciation for his aid and inspiration.

A SYSTEMATIC ANALYSIS OF FREQUENCY DISTRIBUTIONS

BY THE EDGEWORTH METHOD

I. INTRODUCTION

One of the important problems of the statistician is to represent a set of observations or measurements by a concise and suitable formula. This formula may express a physical law or may enable us to represent a wide range of experimental data by a few well-selected constants.

The object of this paper is to analyze and systematize the method of graduating frequency distributions suggested by the late Professor F. Y. Edgeworth, an economist who became interested in statistics. The original idea grew out of Edgeworth's work with the general law of error, which he believed was continually at work in the universe. Edgeworth attempted to represent statistical distributions (from games of chance, errors in observations, physical, biological, and economic data) by mathematical formulas, which were either an approximate expression of the generalized law or some related form.

This paper attempts to develop a system of frequency curves, such as Karl Pearson's System, according to the Method of Translation, a related form.

II. THE METHOD OF TRANSLATION

The Method of Translation consists of the transformation of a normal curve of error by substituting for each of the magnitudes grouped according to such a law some assigned function of that magnitude. For the magnitude ξ grouped according to the normal law, we substitute the magnitude $x = F(\xi)$. If $F(\xi)$ is rapidly convergent, it may be written, with appropriate origin,

$$x = a\xi + k\xi^2 + l\xi^3 \ . \qquad (1)$$

We restrict ourselves to three constants because these, with the mean, make a total of four, and their evaluation by the Method of Moments will require the first four moments. (Higher moments are more liable to error). This is the same reason which prompted Pearson to use four constants in his differential equation.

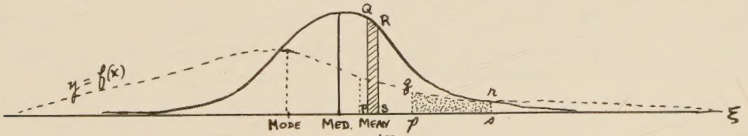
The normal frequency distribution, called the generating curve, may be written

$$\eta \, d\xi = \frac{e^{-\xi^2}}{\sqrt{\pi}} \, d\xi \qquad (2)$$

Using our operator (1), we obtain as the translated curve

$$y \, dx = \eta \, d\xi = \frac{\eta \, dx}{(a + 2k\xi + 3l\xi^2)} = \frac{e^{-\xi^2} dx}{\sqrt{\pi}\,(a + 2k\xi + 3l\xi^2)} \qquad (3)$$

Graphically, we may show the generating curve and the translated curve:



Any area under the normal curve $\eta \, d\xi$ is translated as an area y dx. Every percentile of the ξ-curve becomes the same percentile of the X-curve. The mean of the ξ-curve becomes the median of the X-curve.

The Method of Moments

Let us represent the observed data as the X-curve. The t-th moment of the X-curve about its median, M_t, is obtained by multiplying each element of area y dx by x^t, where x is the deviation of any value X from the median, integrating over the range, and dividing by the total area.

$$M_t = \int_{-\infty}^{\infty} x^t y \, dx = \int_{-\infty}^{\infty} x^t \eta \, d\xi = \int_{-\infty}^{\infty} (a\xi + k\xi^2 + l\xi^3)^t \eta \, d\xi \qquad (4)$$

Since a is never zero in actual work, we may write our operator (1),

$$x = a(\xi + K\xi^2 + \lambda\xi^3) \tag{5}$$

We obtain the following moments about the median:

$$M_1 = \tfrac{1}{2}aK \tag{6}$$

$$M_2 = a^2\left(\tfrac{1}{2} + \tfrac{3K^2}{4} + \tfrac{3\lambda}{2} + \tfrac{15\lambda^2}{2}\right) \tag{7}$$

$$M_3 = a^3\left(\tfrac{9K}{4} + \tfrac{45K\lambda}{4} + \tfrac{15}{8}K^3 + \tfrac{315}{16}K\lambda\right) \tag{8}$$

$$M_4 = a^4\left(\tfrac{3}{4} + \tfrac{45K^2}{4} + \tfrac{15\lambda}{2} + \tfrac{315}{4}K^2\lambda + \tfrac{105}{16}K^4 + \tfrac{315}{8}\lambda^2 + \tfrac{945}{8}\lambda^3 + \tfrac{2835}{16}K\lambda^2 + \tfrac{10395}{64}\lambda^4\right) \tag{9}$$

Transferring to moments about the mean, we have:

$$\mu_2 = \tfrac{a^2}{2}(1 + K^2 + 9\lambda + \tfrac{15}{4}\lambda^2) \tag{10}$$

$$\mu_3 = a^3\left(\tfrac{3K}{2} + K^3 + 9K\lambda + \tfrac{135}{8}K\lambda^2\right) \tag{11}$$

$$\mu_4 = a^4\left(\tfrac{3}{4} + \tfrac{15K^2}{2} + \tfrac{15K^4}{2} + \tfrac{15\lambda}{4} + \tfrac{117}{2}K^2\lambda + \tfrac{315}{8}\lambda^2 + \tfrac{10395}{64}\lambda^4 + \tfrac{1125}{8}K^2\lambda^2 + \tfrac{945}{8}\lambda^3\right) \tag{12}$$

The mean of the X-curve is $\frac{1}{2}aK$ to the right of the median. The constant K is related to the skewness of the X-curve, and λ to the kurtosis.

Keeping in mind the well-known Pearsonian constants, $\beta_1 = \frac{\mu_3^2}{\mu_2^3}$ and $\beta_2 = \frac{\mu_4}{\mu_2^2}$, we use equations (10), (11), and (12), to eliminate a, obtaining, after substituting χ for K^2,

$$\beta_1 = \frac{8\chi\left(\frac{3}{2} + \chi + 9\lambda + \frac{135}{8}\lambda^2\right)^2}{\left(1 + \chi + 3\lambda + \frac{15}{4}\lambda^2\right)^3} \tag{13}$$

$$\beta_2 - 3 = \frac{12\left(2\chi + \lambda + \chi^2 + 18\chi\lambda + 9\lambda^2 + 45\chi\lambda^2 + \frac{135}{4}\lambda^3 + \frac{405}{8}\lambda^4\right)}{\left(1 + \chi + 3\lambda + \frac{15}{4}\lambda^2\right)^2} \tag{14}$$

We must be able to solve equations (13) and (14) for values of λ and χ corresponding to values of β_1 and β_2 which are secured from the observed data.

Since there is no easy recursion formula (as graces the Pearson System) to determine these values, and the method of elimination is highly impracticable, I plotted the values of β_1 and β_2 which were obtained for various given pairs of (λ, χ), and then devised a scheme to read from four maps (two measuring 6x6 feet) the values of λ and χ for any given β_1 and β_2.

The Method of Percentiles

If $\frac{1}{2}-p$ is the proportion of observations above a reading x measured from the median of the observations, then the corresponding value of ξ is given by (5) and

$$p = \int_0^{\xi} \frac{1}{\sqrt{\pi}} e^{-\xi^2} d\xi. \tag{15}$$

The median is not known, but the difference between any two positions on the scale of observations, $x_2 - x_1$, is known, and hence we are able to obtain a number of equations such as

$$x_{t+1} - x_t = a(\xi_{t+1} - \xi_t)\left[1 + K(\xi_{t+1} + \xi_t) + \lambda(\xi_{t+1}^2 + \xi_{t+1}\xi_t + \xi_t^2)\right] \tag{16}$$

from which the required constants, a, λ, and K can be determined.

III. SOME LIMITATIONS

We may set the following limitations on β_1 and β_2:
(a) $\beta_1 \leqq \beta_2$; (b) $1 \leqq \beta_2$; (c) $0 \leqq \beta_1 \leqq 12$, $1 \leqq \beta_2 \leqq 15$.

We prove (a) by showing that the device $\int_{Range}(t^2 + mt)^2 f(t)\,dt$, where $f(t)$ is any frequency function in standard units, and m is a real parameter, is always greater than or equal to zero. Upon integration we obtain the quadratic

$$m^2 + 2\sqrt{\beta_1}\, m + \beta_2 \geqq 0$$

which gives $\beta_1 \leqq \beta_2$.

The proof of (b) follows the same method, with the device $\int_{RANGE}(t^2+m)^2 f(t)dt$.

Theoretically, there is no upper limit on the values of β_1 and β_2. For actual Bell-shaped distributions, among the highest values obtained were those given by Pearson for the Lips of Medusa, $\beta_1 = 4.168$ and $\beta_2 = 12.376$. These correspond to λ and χ values .2 approximately.

For J-curves which present unusual skewness and hence large β_1, the values of χ will be higher. Large positive values of λ show unusual peakedness. Large negative values of λ show an abnormal condition which I call inverse peakedness, characterizing U-shaped curves.

These abnormal curves having so great asymmetry or such unusual kurtosis as to be U-shaped have no evident affinity to the normal curve, but some of the modifications suggested in the form of the normal curve are capable of being applied to them.

For the Edgeworth System, I classify curves with the following ranges:

	β_1	β_2
(1) Normal.	0 to .08	2.96 to 3.04
(2) Subnormal.	.08 to .8	2.6 to 2.96 3.04 to 3.4
(3) Abnormal.	Above .8	Below 2.6 Above 3.4

IV. THE GENERAL CASE

The general equation of the curve obtained by translation of the normal curve by the operator (1) is

$$y = \frac{e^{-\xi^2}}{\sqrt{\pi}(a+2k\xi+3\ell\xi^2)} \tag{17}$$

Taking the derivative with respect to x, we obtain as slope of the tangent to the translated curve at any point x, or corresponding ξ:

$$\frac{dy}{dx} = \frac{e^{-\xi^2}\left[6\ell\xi^3 - 4k\xi^2 - \xi(6\ell+2a) - 2k\right]}{\sqrt{\pi}\ (a+2k\xi+3\ell\xi^2)} \tag{18}$$

The roots of $a+2k\xi+3\ell\xi^2=0$ will give the values of ξ which correspond to the points at which the curve y has infinite ordinates.

We obtain the values of ξ which correspond to the points at which the curve y reaches a maximum or minimum as roots of the cubic equation

$$6\ell\xi^3 + 4k\xi^2 + \xi(6\ell+2a) + 2k = 0 \tag{19}$$

V. SPECIAL CASES

The generating normal curve is not only extended (or contracted), but also in part folded over in the process of translation. For any operator, we attempt to find when $y=0$ and $y=\infty$; when $\frac{dy}{dx}=0$, $\frac{dy}{dx}=\infty$.

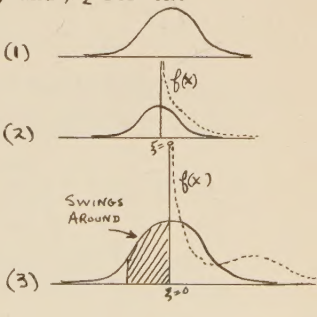
Using equations (13) and (14), we may evaluate β_1 and β_2 for the theoretical distribution. We have the following cases:

(1) Bell-curve. (Normal) $x=a\xi$. $\beta_1=0$, $\beta_2=3$.

(2) J-curve. $x=k\xi^2$. $\beta_1=8$, $\beta_2=15$.

(3) J-curve. $x=k\xi^2+\ell\xi^3$. $\beta_1>8$, $\beta_2>15$

Here we have y infinite for $\xi=0$, $\xi=\frac{-2k}{3\ell}$. The transformed curve loses its typical character of unimodality at the point where $\frac{dy}{dx}$ changes from positive to negative, becoming infinite at $\xi=\frac{-2k}{3\ell}$. (This curve is unimodal if $\left|\frac{\ell}{k}\right| < \frac{1}{3}$).



(4) J-curve. $x = a\xi + k\xi^2$. $\beta_1 = \frac{2(2k^3 + 3a^2k)^2}{(a^2+k^2)^3}$; $\beta_2 = 3 + \frac{12k^2}{a^2+k^2} + \frac{12a^2k^2}{(a^2+k^2)^2}$

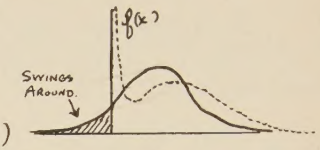
The whole portion of area on the left of the point $\xi = \frac{-a}{2k}$ is swung around to the right.

$\frac{dy}{dx} = \infty$ and $y = \infty$ for $\xi = \frac{-a}{2k}$

$\frac{dy}{dx} = 0$ for $\xi = \frac{-a \pm \sqrt{a^2 - 8k^2}}{4k}$

If $a^2 < 8k^2$, we have a J-curve.

If $a^2 > 8k^2$, we have a curve as shown.



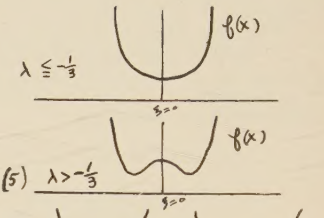
(5) Symmetrical U-curve. $x = a\xi + l\xi^3$ $(l < 0)$

Ordinates infinite for $\xi = \pm\sqrt{\frac{-a}{3l}}$

$\frac{dy}{dx} = 0$ for $\xi = 0$ and $\xi = \pm\sqrt{\frac{-a}{3l} - 1}$

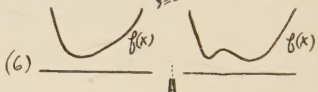
For $\lambda = \frac{l}{a} \leqq -\frac{1}{3}$, we have one inverse mode.

For $0 > \lambda > -\frac{1}{3}$, we have two inverse modes.

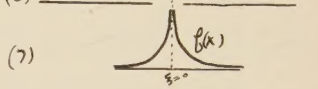


(6) Unsymmetrical U-curve. $x = a\xi + k\xi^2 + l\xi^3$ $(l < 0)$

For $3\lambda^2 + \lambda > \frac{4\chi}{9}$, one inverse mode; for $3\lambda^2 + \lambda < \frac{4\chi}{9}$, two.

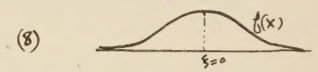


(7) Symmetrical J-curve. $x = l\xi^3$. $\beta_1 = 0$, $\beta_2 = 46.2$.



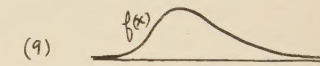
(8) Symmetrical Bell. $x = a\xi + l\xi^3 (l > 0)$.

No infinite ordinates; $\frac{dy}{dx} = 0$ for $\xi = 0, \xi = \pm\infty$.



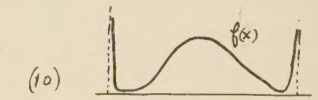
(9) Unsymmetrical Bell. $x = a\xi + k\xi^2 + l\xi^3$.

We get unlimited range for $3\lambda > \chi$.
(This is the criterion for unlimited Bell.)



(10) Limited Bell. $x = a\xi + k\xi^2 + l\xi^3$. $(l > 0)$

The condition $\chi > 3\lambda$ gives two infinite ordinates. If the cubic (19) has three roots, we obtain a curve as shown.

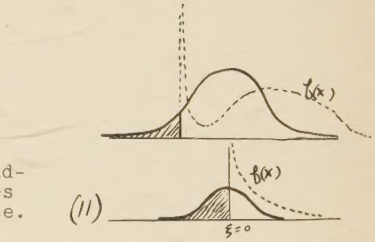


(11) J-curve. $x = a\xi + k\xi^2 + l\xi^3$.

The criterion for this type is $\chi = 3\lambda$. We obtain limited range in one direction for $\lambda > 0$. The point $\xi = \frac{-k}{3l}$ gives an infinite ordinate.

The area to the left of the point is revolved about the axis to give the curve shown.

The curve will be unimodal or bimodal, depending on whether $\frac{dy}{dx}$ changes sign as ξ approaches the point at which the curve becomes infinite.



VI. THREE TYPES OF CURVES IN THE EDGEWORTH SYSTEM

Since we never have $a = 0$ in application, we use the λ and χ notation.

We classify curves in the Edgeworth System in three categories:

1. Bell-curve.
   (a) Unlimited range-- $0 < \chi < 3\lambda$.
   (b) Limited range -- $0 < 3\lambda < \chi$, or $\lambda = 0, \chi = \frac{1}{8}$.
2. U-curve. $\lambda < 0$.
3. J-curve. $\lambda = 0$, or if $\lambda > 0$, $\chi = 3\lambda$.

For $\lambda < 0$, the curve becomes abrupt at the ends, but it may still be similar to the Bell-curve.

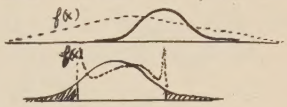
When $\lambda > 0$, we have no difficulty with actual graduation of data. For each x we obtain a single value of ξ. For $\lambda < 0$, we obtain three values of ξ for each x. This shows limited range.

The curve has no finite ends, one, or two finite ends, according as $\chi \lessgtr 3\lambda$.

Actually the U-shaped curves are merely inverted Bell-curves (with limited range), but I make the distinction in the Edgeworth System because there is a very special difference in λ for the two types.

Bell-curve (Unlimited range). No infinite ordinates. $\chi < 3\lambda$.
J-curve. One infinite ordinate. Either $\lambda = 0$ or $\chi = 3\lambda$. (Included in this category are what may be called one-finite-end Bell.)
U-curve. Two infinite ordinates. $\lambda < 0$.

Unlimited range means that the normal curve is simply translated, without any folding over. Limited range means that the tails of the normal curve are folded over. For each x there are three values of ζ. Some of these may be so far from the center of the generating normal curve that areas in their vicinity are negligible.



The criterion for the number of maximum or minimum points is the number of roots of the cubic (19), one real root or three, according as

$$3\lambda^2 + \lambda \gtrless \frac{4\chi}{9}$$

For the J-curve, if $\lambda = 0$, we have either no apparent maximum, or two points at which (19) equals zero, according as $\chi \gtrless \frac{1}{3}$. This latter case will be at times our Bell-curve with range limited in one direction.

VII. COMPARISON WITH THE PEARSON SYSTEM

The changes in the magnitudes of λ and χ, and the sign of λ, suffice in the ordinary (practical) cases to represent the data. By considering these changes we reproduce all the principal variations of the Pearson System, namely:

(1) Limited range in both directions--Pearson Types I,II,VI.
(2) Limited range in one direction --Pearson Types III, V.
(3) Unlimited range --Pearson Types IV, VII.

The Type III curve which does not use the fourth moment is the dividing line between two types which use the fourth moment, range for one type limited in both directions, for the other type unlimited.

The case in the Edgeworth System where the fourth moment is not required is that in which $\lambda = 0$.

We find the maximum point for the translated curve by equating (19) to zero, $\zeta - 2\kappa\zeta^2 - \kappa = 0$. Substituting $\zeta = -2\kappa\zeta^2 - \kappa$ in the operator $x = a(\zeta + \kappa\zeta^2)$, we obtain

$$x = a(-\kappa\zeta^2 - \kappa).$$

This represents the mode of the X-curve. We find its distance from the median ($\zeta = 0$ is the median of both normal and X-curve) to be $-a\kappa$.

Since the mean of the X-curve is $\frac{1}{2}a\kappa$ to the right of the median, we get the relation which Pearson obtained for the Type III curve,

$$\text{Mean} - \text{Median} = \tfrac{1}{3}(\text{Mean} - \text{Mode}).$$

VIII. A SET OF EXAMPLES

I graduated by the Edgeworth System the following Pearson Types as given and graduated by W. Palin Elderton[2] in "Frequency Curves and Correlation": I, II, III, IV, V, VI, VII, and X. The "fits" were unusually good, in many cases better than those by the Pearson method.

I present in order the procedure for carrying out an actual graduation of observed frequency data:

1. Calculate $\nu_1, \nu_2, \nu_3, \nu_4$, moments about the mean of the data.
2. Calculate β_1 and β_2 for the data. (Consider ν_1's equal to μ_1's.)
3. Use author's Table 1 to find λ, Table 2 for χ, corresponding to given or calculated β_1 and β_2.
4. Use equation $\mu_2 = \frac{a^2}{2}(1 + \chi + 3\lambda + \frac{15}{4}\lambda^2)$ to find the value of a.

5. Evaluate $K = \pm\sqrt{\chi}$. Find distance of median from computed mean $= \frac{1}{2}aK$.
6. Find median by subtracting $\frac{1}{2}aK$ from the computed mean.
   (This is the center of the generating normal curve.)
7. Find deviations of class boundaries from the median, x_n's.
8. Divide all deviations by a.
9. Write the equation of translation operator, $\lambda\xi^3 + K\xi^2 + \xi = \frac{x}{a}$

We now have to find values of ξ corresponding to every computed $\frac{x}{a}$. This involves the solution of a cubic equation. Considering the cubic equation as

$$a_0 x^3 + 3a_1 x^2 + 3a_2 x + a_3 = 0,$$

we have $a_0 = \lambda$; $a_1 = \frac{K}{3}$; $a_2 = \frac{1}{3}$; and $a_3 = \frac{-x}{a}$.

10. Compute $H = a_0 a_2 - a_1^2$. If $H > 0$, we have only one real value of ξ for each $\frac{x}{a}$; if $H < 0$, we have three real values of ξ for each $\frac{x}{a}$.
11. Multiply each value of $\frac{x}{a}$ by $-a_0^2$.
12. To each value so obtained, add $p = -3a_0 a_1 a_2 + 2a_1^3$. We call this value G. The H and G are the values in the equation $y^3 + 3Hy + G = 0$, to which we have reduced our original cubic by transformation $x = \frac{1 - a_1}{a_0}$.
13. Compute $2H\sqrt{-H}$ if $H < 0$; $2H\sqrt{H}$ if $H > 0$.
14. Divide the set of G values by the constant $2H\sqrt{\pm H}$.
    These represent $\sinh 3z$ if $H > 0$; or $\cos 3\theta$ if $H < 0$.
15. Using proper tables, find 3z or 3θ, corresponding to those values.
16. Divide the values obtained by 15. by 3, securing set of z's or θ's.
17. For the set of z's simply find $\sinh z$. For the set of θ's, find $\cos\theta$, $\cos(\theta + 120^\circ)$, and $\cos(\theta + 240^\circ)$.
18. Evaluate $k = 2\sqrt{\pm H}$. Multiply each value obtained by 17. by k.
19. Subtract from each value obtained by 18. the quantity a_1.
20. Divide each value so obtained by a_0.

We now have the value or values of ξ corresponding to each $\frac{x}{a}$. If we are using a set of tables of the normal function written $\frac{1}{\sqrt{2\pi}} e^{-\xi^2}$, we simply find the area given by the table for the nrrmal curve between two values of ξ. Multiplying this value by N, we have the graduated frequency.

If we are using a set of tables of the normal function written $\frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, then we must first multiply each value of ξ by $\sqrt{2}$. We then find the area under the curve between two values $\sqrt{2}\,\xi$. Multiplying by N, we have graduated frequency.

In the event we have three values of ξ for each $\frac{x}{a}$, we may neglect all values of ξ or $\sqrt{2}\,\xi$, depending on tables used, which exceed 3 or 4 numerically. We add the areas under the curve for the values $\xi_1 - \xi_2$, $\xi_1' - \xi_2'$, $\xi_1'' - \xi_2''$, corresponding to $\frac{x_1}{a} - \frac{x_2}{a}$. Multiply by N to get the graduated frequency.

IX. SUMMARY AND CONCLUSION

Starting with the simple notion of translation, I have developed a system whereby frequency distributions may be classified. I have given the criteria which characterize each type of frequency distribution according to this method.

I have systematized a procedure to enable anyone to use the Method of Translation to graduate frequency distributions. To encourage the use of this method, I have tabled the values of λ and χ, corresponding to pairs of β_1, β_2.

After graduation of eight Pearson Types (both by the Pearson System[3] and the Edgeworth System), I point out the advantages the Edgeworth System has over the Pearson System:

1. The classification of curves is simpler. Curves are either Bell, J- or U-curves, limited or unlimited range. Confusion attending a large number of types is missing.

2. A variety of class intervals may be used, such as percentile graduations. This cannot be done with the Pearson System.

3. The method has wide application. The only difficulty is that for very pronounced J- or U-distributions, finding values of λ and $\varkappa$. This difficulty, together with the fact that the standard error of the median for U-distributions is larger than that for the median of Bell-curves, suggests a method of inversion of the U-curve into a Bell-curve.

4. The Edgeworth System requires only two sets of tables, one for the areas under the normal frequency function, and the other, the values of λ and $\varkappa$ corresponding to β_1 and β_2, which I have computed and tabled.

5. The method of application requires a single form and a single procedure in the Edgeworth System in contrast to the variety of forms employed in the Pearson System.

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